Assessment Schedule – 2005

Calculus: Manipulate real and complex numbers, and solve equations (90638)

Evidence Statement

	Achievement Criteria	Q	Evidence	Code	Judgement	Sufficiency
Achievement	Manipulate real and complex numbers and solve equations.	1(a)	$\overline{5+7\text{i}} - 2(3-4\text{i})$ $= 5 - 7\text{i} - 6 + 8\text{i}$ $= -1 + \text{i}$	A1	No alternative.	**Achievement:** 3 × code A1 **and** 2 × code A2. Only 1 × code A2 available from question 6. No repeated skills.
		1(b)	$20\text{cis}\dfrac{5\pi}{12}$	A1	Or equivalent.	
		1(c)	$\dfrac{69+46\sqrt{3}}{-23}$ $= -3 - 2\sqrt{3}$	A1	No alternative.	
		1(d)	$(2\text{cis}\dfrac{\pi}{6})^5$ $= 32\text{cis}\dfrac{5\pi}{6}$ $= -16\sqrt{3} + 16\text{i}$	A1	Or equivalent.	
		2(a)	$x = 3 \pm 2\sqrt{5}\,\text{i}$	A2	Or equivalent.	
		2(b)	$(2x-1)(x+2)(x-2) = 0$ $x = 0.5, -2, 2$	A2	Must factorise. No alternative.	
		2(c)	$2^{x-1} = 7$ $x = 3.81$ (3 sf)	A2	Or equivalent.	

	Achievement Criteria	Q	Evidence	Code	Judgement	Sufficiency
Achievement with Merit	Solve more complicated equations.	3	$\ln(5x-3)-\ln(x+1)=\ln p$ $\ln\frac{5x-3}{x+1}=\ln p$ $5x-3=px+p$ $5x-px=p+3$ $x=\frac{p+3}{5-p}$	A2 M	Or equivalent	**Achievement with Merit:** **EITHER** As for Achievement **plus** 2 × code M **OR** 3 × code M Only 1 × code M available from question 6.
		4	$r^3\operatorname{cis}3\theta=125\operatorname{cis}\pi$ $r^3=125$ $r=5$ $3\theta=\pi+k.2\pi$ $\theta=\frac{\pi}{3}+k.\frac{2\pi}{3}$ $k=0 \quad \theta=\frac{\pi}{3}$ $k=1 \quad \theta=\pi$ $k=-1 \quad \theta=-\frac{\pi}{3}$ $5\operatorname{cis}\frac{\pi}{3},\ 5\operatorname{cis}\pi,\ 5\operatorname{cis}(-\frac{\pi}{3})$	A1 A2 M	Conversion to polar form. Or equivalent.	
		5	$(\sqrt{x-k})^2=(\sqrt{x}-2)^2$ $x-k=x-4\sqrt{x}+4$ $4\sqrt{x}=k+4$ $16x=k^2+8k+16$ $x=\frac{1}{16}(k^2+8k+16)$	A2 M	Or equivalent	

	Achievement Criteria	Q	Evidence	Code	Judgement	Sufficiency
Achievement with Excellence	Solve problem(s) involving real or complex numbers.	6	$z^4 = 1$ $z^2 = \pm 1$ $z = \pm 1, \pm \mathrm{i}$ The four roots of 1: 1, i, –1, –i	A2	No alternative.	**Achievement with Excellence:** As for Merit **plus** code E.
		*	$z^2 + 1 = 1$ $z = 0$			
		*	$z^2 + 1 = \mathrm{i}$ $z^2 = -1 + \mathrm{i}$ $z^2 = \sqrt{2}\,\mathrm{cis}\,\frac{3\pi}{4}$ $r^2 \mathrm{cis}\,2\theta = \sqrt{2}\,\mathrm{cis}\,\frac{3\pi}{4}$ $r = \sqrt[4]{2} \qquad 2\theta = \frac{3\pi}{4} + k.2\pi$ $\theta = \frac{3\pi}{8}, -\frac{5\pi}{8}$ $z = \sqrt[4]{2}\,\mathrm{cis}\,\frac{3\pi}{8},\ \sqrt[4]{2}\,\mathrm{cis}\left(-\frac{5\pi}{8}\right)$	M	Or equivalent.	
		*	$z^2 + 1 = -1$ $z^2 = -2$ $z = \sqrt{2}\mathrm{i}, -\sqrt{2}\mathrm{i}$	A2	Or equivalent.	
		*	$z^2 + 1 = -\mathrm{i}$ $z^2 = -1 - \mathrm{i}$ $z^2 = \sqrt{2}\,\mathrm{cis}\left(-\frac{3\pi}{4}\right)$ $r^2 \mathrm{cis}\,2\theta = \sqrt{2}\,\mathrm{cis}\left(-\frac{3\pi}{4}\right)$ $r = \sqrt[4]{2}$ $2\theta = -\frac{3\pi}{4} + k.2\pi$ $\theta = -\frac{3\pi}{8}, \frac{5\pi}{8}$ $z = \sqrt[4]{2}\,\mathrm{cis}\,(-\frac{3\pi}{8}),\ \sqrt[4]{2}\,\mathrm{cis}\,\frac{5\pi}{8}$	M	Or equivalent.	
			There are seven solutions.	E	Must have 5 or more solutions.	

Judgement Statement

Achievement	Achievement with Merit	Achievement with Excellence
Manipulate real and complex numbers, and solve equations. 3 × A1 *and* 2 × A2 (only one A2 can be from Q6)	Solve more complicated equations. Achievement *plus* 2 × M ***or*** 3 × M (Only one M can be from Q6)	Solve problem(s) involving real or complex numbers. Merit *plus* 1 × E